Energy-Efficient Resource Allocation for Heterogeneous Cognitive Radio Networks with Femtocells

Renchao Xie†, F. Richard Yu†, and Hong Ji∗

∗Key Lab. of Universal Wireless Comm., Ministry of Education, Beijing University of Posts and Telecom., Beijing, P.R. China
†Depart. of Systems and Computer Eng., Carleton University, Ottawa, ON, Canada
Email: rxie@sce.carleton.ca, richard_yu@carleton.ca, jihong@bupt.edu.cn

Abstract—Both cognitive radio and femtocell have been considered as promising techniques in wireless networks. However, most of previous works are focused on spectrum sharing and interference avoidance, and the energy efficiency aspect is largely ignored. In this paper, we study the energy efficiency aspect of spectrum sharing and power allocation in heterogeneous cognitive radio networks with femtocells. To fully exploit the cognitive capability, we consider a wireless network architecture in which both the macrocell and the femtocell have the cognitive capability. We formulate the energy-efficient resource allocation problem in heterogeneous cognitive radio networks with femtocells as a Stackelberg game. A gradient based iteration algorithm is proposed to obtain the Stackelberg equilibrium solution to the energy-efficient resource allocation problem. Simulation results are presented to demonstrate the Stackelberg equilibrium is obtained by the proposed iteration algorithm and energy efficiency can be improved significantly in the proposed scheme.

I. INTRODUCTION

Rapidly rising energy costs and increasingly rigid environmental standards have led to an emerging trend of addressing “energy efficiency” aspect of wireless communication technologies [1]. In a typical wireless cellular network, the radio access part accounts for up to more than 70 percent of the total energy consumption [2]. Therefore, increasing the energy efficiency of radio networks is very important to meet the challenges raised by the high demands of traffic and energy consumption.

Cognitive radio technology can play an important role in improving energy efficiency in radio networks [3]. The cognitive abilities have a wide range of properties, including spectrum sensing [4], spectrum sharing [5] and adaptive transmission [6], which are beneficial to improve the tradeoff among energy efficiency, spectrum efficiency, bandwidth, and deployment efficiency in wireless networks [2].

Some works have been done to consider energy efficiency in cognitive radio networks. In [7], authors study the hierarchy in energy games for cognitive radio networks. Authors of [8] study the distributed power control game to maximize the transmission energy-efficiency for SUs in cognitive radio networks. Energy-efficient power control and receiver design in cognitive radio networks are studied in [9].

On the other hand, femtocell has been considered as a promising technique, and has been integrated in current and future radio access networks [10]. Due to its short transmit-receive distance property, femtocell technique can reduce energy consumption, prolong handset battery life, and increase network coverage [11]. The problem of energy efficiency for femtocell base station is studied in [12], where a novel energy saving procedure is designed for femtocell base stations. In [13], resource sharing and access control in OFDMA femtocell networks are studied, where users’ selfish characteristics are considered and an incentive mechanism is designed for subscribers to share the resource of femtocell base stations.

Combining cognitive radio with femtocell can further improve the system performance [14]–[17]. In [15], a femtocell-based cognitive radio architecture is designed to enable multi-tiered opportunistic access in wireless networks. The smart cognitive femtocell is considered in [16]. Authors of [17] study the problem of downlink spectrum sharing for cognitive radio femtocell networks, where the mixed primal and dual decomposition methods are proposed to realize the downlink spectrum sharing to maximize the system capacity.

Although some works have been done for heterogeneous cognitive radio networks with femtocells, most of previous works are focused on spectrum sharing and interference avoidance. Consequently, the energy efficiency aspect in this setting is largely ignored. In addition, most of previous works assume that only the femtocell base station has the cognitive capability, without considering the cognitive capability of the macrocell base station.

In this paper, our work is different from previous works.
We focus on the energy efficiency aspect of spectrum sharing and power allocation in heterogeneous cognitive radio networks with femtocells. Since both cognitive radio and femtocell are promising technologies to enable energy efficiency in wireless networks, the interplay between them merits further research. We use bits/Hz per Joule [18] to measure the performance of energy efficiency. To fully exploit the cognitive capability, we consider a wireless network architecture in which both the macrocell and the femtocell have the cognitive capability. This network architecture is first proposed in [14], which is of importance in practical cellular femtocell networks, where the macrocell base stations can sense the TV band. We formulate the energy-efficient resource allocation problem in heterogeneous cognitive radio networks with femtocells as a Stackelberg game. Then a gradient based iteration algorithm is proposed to obtain the Stackelberg equilibrium to the energy-efficient resource allocation problem.

The rest of this paper is organized as follows. In Section II, the system model is given, and the Stackelberg game model is formulated. In Section III, the Stackelberg equilibrium solution is presented. Simulation results are presented and discussed in Section IV. Finally, we conclude this study in Section V.

II. SYSTEM DESCRIPTION

A. System Model

We consider a communication system that consists of primary networks and a femtocell-based heterogeneous cognitive radio network, as shown in Fig. 1. A primary network can sell part of spectrum resource to the heterogeneous cognitive radio network with femtocells to earn additional profit. In the heterogeneous cognitive radio network, there are multiple macro secondary users (MSUs), a cognitive base station, multiple femtocells, and multiple femtocell secondary users (FSUs). The cognitive base station allocates the spectrum resource bought from the primary networks to femtocells or MSUs directly based on the channel quality to maximize its revenue. In each femtocell, there is a femtocell base station (FBS) to provide service to FSUs, where the FBS is connected to the cognitive base station over a broadband connection, such as cable modem or digital subscriber line (DSL) [11]. The whole system is operated in a time-slotted manner, and the primary networks and the femtocell-based cognitive radio network are assumed to be perfectly synchronized. To simplify the analysis of the problem, without loss generality, we assume that there is only one FSU that is serviced by the FBS in each time slot, which is commonly assumed in the literature due to the opportunistic scheduling operation in practice [19].

Under this framework, we assume that there are $P$ primary networks. Each primary network $p$ is willing to offer a spectrum selling price $c_p$ and sells its part spectrum resource $w_p$ of total spectrum $W_p$ to the heterogeneous cognitive radio network to maximize its profit. The cognitive base station can buy the spectrum resource $w_p$ from primary network $p$ depending on the spectrum price. Then the cognitive base station allocates the spectrum $w_p$ to the femtocells or MSUs directly to gain its revenue. Here, we assume that the spectrum resource $w_p$ bought from a certain primary network is only allocated to one femtocell or MSU. Assume there are $K_{tot}$ femtocells and $I_{tot}$ MSUs directly served by the cognitive base station in the heterogeneous cognitive radio network. Usually the total number of femtocells and MSUs requesting to connect to the cognitive base station is large, that is, $(K_{tot} + I_{tot}) \geq P$. Due to the limited resource, there are at most $P$ of $(K_{tot} + I_{tot})$ femtocells and MSUs that can access to the cognitive base station in each time slot, which can be realized by scheduling and access control [20]. Here, without loss generality, we assume $K$ femtocells and $J$ MSUs access the cognitive base station, where $K + I = P$. In each time slot, when a certain spectrum $w_p$ is used by a femtocell, the aim of the femtocell is to maximize its energy-efficient communications by allocating its power. Similarly, when the spectrum is used by MSUs, the cognitive base station performs the energy-efficient power allocation. Based on the discussion above, we can formulate the problem of resource allocation for energy-efficient communications in the heterogeneous cognitive radio network as a three-stage Stackelberg game problem, as described in the following subsection.

B. Problem Formulation

In this subsection, we formulate the energy-efficient resource allocation problem as a three-stage Stackelberg game, as shown in Fig. 2, which consists of leaders and followers. Here, we view the up-stage as the leaders that move first,
where $\pi_k$ denotes the additional circuit power consumption of devices during transmissions (e.g., digital-to-analog converters, filters, etc.), which is independent to the data transmission power. $h_{pk}$ and $p_k$ are the channel gain and power allocation on spectrum $w_p$ for FBS $k$, respectively. $\sigma^2$ is the additive Gaussian white noise with zero mean and unit variation. In (1), bits/Hz per Joule is used as the energy efficiency metric, which is widely used in wireless networks [18]. To improve energy efficiency, less energy should be used to transmit more information data. Based on (1), the aim of FBS $k$ is to maximize its utility given spectrum $w_p$, which can be expressed as follows.

$$\eta_{pk} = \log_2 \left( 1 + \frac{h_{pk}^2 p_k}{\sigma^2} \right),$$

(1)

where $p_a$ denotes the additional circuit power consumption of devices during transmissions (e.g., digital-to-analog converters, filters, etc.), which is independent to the data transmission power. $h_{pk}$ and $p_k$ are the channel gain and power allocation on spectrum $w_p$ for FBS $k$, respectively. $\sigma^2$ is the additive Gaussian white noise with zero mean and unit variation. In (1), bits/Hz per Joule is used as the energy efficiency metric, which is widely used in wireless networks [18]. To improve energy efficiency, less energy should be used to transmit more information data. Based on (1), the aim of FBS $k$ is to maximize its utility given spectrum $w_p$, which can be expressed as follows.

$$\pi_k = \sum_{p=1}^{P} x_{pk} (s_k w_p \eta_{pk} - c_k w_p \eta_{pk}) = \sum_{p=1}^{P} (s_k - c_k) x_{pk} w_p \eta_{pk},$$

(2)

where $w_p$ is the spectrum resource bought from primary network $p$, $c_k$ is the revenue for FBS $k$; and $c_k$ is the cost charged by the cognitive base station due to the allocated spectrum. $x_{pk} \in \{0, 1\}$ is the spectrum allocation index, where $x_{pk} = 1$ means that the spectrum bought from primary network $p$ is allocated to FBS $k$; otherwise $x_{pk} = 0$. Here, we have $s_k > c_k$. Otherwise, the FBSs will not request to access. When the FBSs finish the energy-efficient power allocation, there is feedback information to the cognitive base station. For the cognitive base station, the aim is to buy the size of spectrum from primary networks and allocate them to the FBSs or MSUs directly to maximize its revenue. Here, we notice that the cognitive base station also does the energy-efficient power allocation for MSUs when the spectrum is allocated to MSUs, which is similar to (1). Therefore, we can use the following quadratic utility function for cognitive base station.

$$\pi_b (w) = \sum_{p=1}^{P} w_p \left( \sum_{k=1}^{K} c_k w_p \eta_{pk} + \sum_{i=1}^{I} \xi_i x_{pi} \eta_{pi} \right) - \frac{1}{2} \left( \sum_{p=1}^{P} w_p^2 + 2\theta \sum_{p \neq p} w_p w_q \right) - \sum_{p=1}^{P} c_p w_p,$$

(3)

where $w = \{ w_1, w_2, ..., w_P \}$ denotes the spectrum bought from primary networks, $\eta_{pi}$ is the energy-efficient transmission parameter for MSUs, $\xi_i$ is the cost paid by MSUs to the cognitive base station, $c_p$ is the price offered by primary network $p$, and $\theta \in [-1, 1]$ is the spectrum substitutability parameter. $\theta = 1$ means that the FBSs or MSUs can switch among the spectrum; $\theta = 0$ means that FBSs or MSUs cannot switch among the spectrum; $\theta < 0$ means spectrum used by FBSs or MSUs is complementary. $x_{pi} \in \{0, 1\}$ is the spectrum allocation index, where $x_{pi} = 1$ means that the spectrum bought from primary network $p$ is allocated to MSU $i$; otherwise $x_{pi} = 0$. Here, to avoid the interference, we assume that one spectrum resource from a certain primary network could only be allocated to one FBS or MSU, i.e.,

$$\sum_{k=1}^{K} x_{pk} + \sum_{i=1}^{I} x_{pi} = 1.$$  

(4)

The motivations to use the utility function in (3) mainly are as follows. First, the utility function is concave about the spectrum demand, which is easy to denote the maximum revenue of cognitive base station. Second, we can get a linear spectrum demand function by differentiating the utility function (3).

For the primary networks, they can sell part of spectrum to the cognitive base station to gain additional profit. Therefore, the aim is to maximize its revenue by offering the spectrum price $c_p$ depending on the spectrum demand from the cognitive base station. We can adopt the following utility function to denote the revenue of primary network $p$.

$$\pi_p (c) = \alpha_1 (W_p - w_p) \kappa_p + c_p w_p,$$

(5)

where $\alpha_1$ is a positive design parameter, which denotes the weight for the revenue from the PUs, $W_p$ is the total spectrum licensed to primary network $p$, and $\kappa_p$ denotes the spectrum transmission efficiency of primary network $p$. Usually, we have $c_p \geq \alpha_1 \kappa_p$, otherwise, the primary networks is not willing to sell its spectrum to the heterogeneous cognitive radio network. The intuitive explanation of the utility function (5)
is that the primary networks will earn the additional revenue from cognitive networks by selling part of its spectrum at the cost of degradation its communication performance.

III. STACKELBERG EQUILIBRIUM SOLUTION

In this section, we use the backward induction method to solve the Stackelberg equilibrium for the three-stage Stackelberg game formulated above.

A. Energy-Efficient Power Allocation for FBSs

When spectrum $w_p$ is allocated to FBS $k$, FBS $k$ aims to maximize its utility function (2) to realize energy-efficient power allocation. However, utility function (2) is a nonlinear function about the transmission power, which makes it hard to directly solve (2). Therefore, to reduce the computation complexity, we propose a gradient assisted binary search algorithm to realize the energy-efficient power allocation.

Before giving the algorithm, we first prove $\pi_k (p_k)$ is a quasiconcave function and give the definition of quasiconcave function as follows.

**Definition 1:** A mapping function $f : \Omega \to \mathbb{R}$, defined on a convex set $\Omega$ of real $n$-dimensional vectors, is strictly quasiconcave, if, for all $x, y \in \Omega$, $x \neq y$ and $0 < \lambda < 1$, we have $f(\lambda x + (1 - \lambda)y) > \min \{f(x), f(y)\}$.

Let

$$R_k (p_k) = \sum_{p=1}^{P} \left( (c_k - c_p) x_{pk} w_pk \log_2 \left( 1 + \frac{h_{pk}^2 R_k}{\sigma^2} \right) \right).$$

(6)

Therefore, based on the above definition, we can give the following properties for $\pi_k (p_k)$.

**Lemma 1:** If $R_k (p_k)$ is strictly concave in $p_k$, $\pi_k (p_k)$ is strictly quasiconcave. Furthermore, $\pi_k (p_k)$ is either first strictly increasing and then strictly decreasing in any $p_k$ or strictly decreasing.

Inspired by [21], we can prove Lemma 1 as follows.

**Proof:** Denote the $\alpha$-sublevel sets of function $\pi_k (p_k)$ as

$$S_\alpha = \{ p_k > 0 | \pi_k (p_k) \geq \alpha \}$$

(7)

Based on the propositions, $\pi_k (p_k)$ is strictly quasiconcave if and only if $S_\alpha$ is strictly convex for all $\alpha$. In this case, when $\alpha < 0$, there are no points satisfying $\pi_k (p_k) = \alpha$. When $\alpha = 0$, only $p_k = 0$ satisfies $\pi_k (0) = \alpha$. Therefore, $S_\alpha$ is strictly convex when $\alpha \leq 0$. When $\alpha > 0$, we can rewrite the $S_\alpha$ as $S_\alpha = \{ p_k > 0 | \alpha (p_a + p_k) = R_k (p_k) \leq 0 \}$. Since $R_k (p_k)$ is strictly concave in $p_k$, which means that $-R_k (p_k)$ is strictly convex in $p_k$, therefore $S_\alpha$ is also strictly convex. Hence, $\pi_k (p_k)$ is strictly quasiconcave function.

Next, we can obtain the derivative of $\pi_k (p_k)$ with $p_k$ as

$$\frac{\partial \pi_k (p_k)}{\partial p_k} = \frac{R_k' (p_k) (p_a + p_k) - R_k (p_k)}{(p_a + p_k)^2} = \frac{\phi (p_k)}{(p_a + p_k)^2},$$

(8)

where $R_k' (p_k)$ denotes the first derivative $\frac{\partial R_k (p_k)}{\partial p_k}$. Based on the above Lemma, if there is $p_k^* \text{satisfying } \frac{\partial \pi_k (p_k)}{\partial p_k} \bigg|_{p_k=p_k^*} = 0$, $p_k^*$ is unique. Now we only need to solve the problem when $p_k^*$ exists.

The derivative of $\phi (p_k)$ is

$$\phi' (p_k) = R_k'' (p_k) (p_a + p_k),$$

(9)

where $R_k'' (p_k)$ is the second derivative of $R_k (p_k)$ with respect to $p_k$. Due to $R_k'' (p_k) < 0$, $\phi' (p_k) < 0$. Therefore, $\phi (p_k)$ is strictly decreasing. In addition, according to the L’Hospital’s rule [21], we know that

$$\lim_{p_k \to \infty} \phi (p_k) = \lim_{p_k \to \infty} \left( R_k' (p_k) (p_a + p_k) - R_k (p_k) \right)$$

$$= \lim_{p_k \to \infty} \left( \frac{R_k (p_k) (p_a + p_k) - R_k (p_k)}{p_k} \right)$$

$$= \lim_{p_k \to \infty} \left( \frac{R_k' (p_k) (p_a + p_k) - R_k (p_k)}{p_k} \right) < 0$$

(10)

and

$$\lim_{p_k \to 0} \phi (p_k) = \lim_{p_k \to 0} \left( R_k' (p_k) (p_a + p_k) - R_k (p_k) \right)$$

$$= R_k' (p_k^0) (p_a + p_k^0) - R_k (p_k^0),$$

(11)

where $p_k^0 = 0$. Hence, we can get the following two cases:

1. When

$$R_k' (p_k^0) (p_a + p_k^0) - R_k (p_k^0) \geq 0, \lim_{p_k \to 0} \phi (p_k) \geq 0.$$

2. When

$$R_k' (p_k^0) (p_a + p_k^0) - R_k (p_k^0) < 0, \lim_{p_k \to 0} \phi (p_k) < 0.$$

Together with (10), we can observe that $p_k^* \text{exists and } \pi_k (p_k)$ is first strictly increasing and then strictly decreasing in $p_k$.

We know that if there is a local maximum solution for the quasiconcave functions, the solution is also globally optimal. Hence, we can give Theorem 1 according to the proof of Lemma 1.

**Theorem 1:** If $R_k (p_k)$ is strictly concave, there exists a unique globally optimal power allocation $p_k^*$, for (2), where $p_k^*$ is obtained by

1. When $\frac{\partial R_k (p_k)}{\partial p_k} \bigg|_{p_k=p_k^*} \geq \frac{R_k (p_k^0)}{(p_a + p_k^0)}, \frac{\partial \pi_k (p_k)}{\partial p_k} \bigg|_{p_k=p_k^*} = 0.$

2. When $\frac{\partial R_k (p_k)}{\partial p_k} \bigg|_{p_k=p_k^*} < \frac{R_k (p_k^0)}{(p_a + p_k^0)}, p_k^* = 0.$

Therefore, based on Theorem 1, we can solve the nonlinear equation to find the optimal power allocation solution or we can use the low-complexity iterative algorithms based on the
obtained by the cognitive base station, which will be shown to allocate the spectrum to FBSs or MSUs with higher
base station, for the primary networks, the revenue of each

2). Do repeat as (13) at the top of the next page.

1). Initialization: the set of FBSs \( \Omega_k \) and the set of primary networks \( \Omega_P \).

Thus, by solving (12), we can obtain the size of spectrum bought from primary networks \( p \) for the cognitive base station as (13) at the top of the next page.

Based on (13), the cognitive base station can maximize its utility function by buying the spectrum from primary networks and allocating the spectrum to FBSs or MSUs.

Here, we must notice that for the different spectrum allocation \( x_{pk}, x_{pi} = \sum_k x_{pk} + \sum_i x_{pi} = 1 \), to FBSs or MSUs, we could get the different size of spectrum demand \( w_p^* \) in (13) and obtain the different revenue \( \pi_b(w) \) of the cognitive base station in (3). Hence, how to allocate the spectrum to FBSs or MSUs is an important problem. We can use the following spectrum allocation algorithm.

**Spectrum Allocation Algorithm to FBSs or MSUs**

1). Initialization: the set of FBSs \( \Omega_k \) = \{1, 2, ..., \( K \)\},

MSUs \( \Omega_i \) = \{1, 2, ..., \( I \)\} and the set of primary networks \( \Omega_P \) = \{1, 2, ..., \( P \)\}.

2). Do repeat

Find \( v^* = \max \sum_k c_k \eta_{pk} \xi_i \eta_{pi} \), then let \( x_{pu} = 1 \),

\( \Omega_k = \Omega_k - v^* \) or \( \Omega_i = \Omega_i - v^* \). And let \( \Omega_p = \Omega_p - p \);

Until the \( \Omega_p = \emptyset \), end repeat.

3). Output the spectrum allocation index \( x_{pk}, x_{pi} \).

In the spectrum allocation algorithm above, when the spectrum is allocated to MSUs, the cognitive base station can do the energy-efficient power allocation for MSUs as described in Subsection III-A. The intuitive explanation of the above algorithm is that the cognitive base station expects to allocate the spectrum to FBSs or MSUs with higher energy-efficient transmission, then more revenue could be obtained by the cognitive base station, which will be shown in the simulation section.

**C. Prices Determination for the Primary Networks**

After the spectrum demand is derived by the cognitive base station, for the primary networks, the revenue of each primary network is affected not only by its price \( c_p \) but also by the prices \( c_{-p} \) offered by other primary networks, where \( c_{-p} = (c_1, ..., c_{p-1}, c_{p+1}, ..., c_P) \) are the prices offered by all the primary networks except \( p \). Therefore, the prices determination among the primary networks is a price competitive game \( G = \{\mathcal{N}, \{c_p\}, \{\pi_p\}\} \), where \( \mathcal{N} = \{1, 2, ..., P\} \) is the set of players, \( c_p \) is the strategy set and \( \pi_p(\cdot) \) is the payoff function of primary network \( p \). Hence, the Nash equilibrium of a game is a strategy equilibrium that no primary network can increase its revenue by choosing a different strategy, given other players’ strategy [22]. In this case, we can use the best response function to find the Nash equilibrium. When the other’s strategy \( c_{-p} \) is given, the best respond function of primary network \( p \) can be defined as follows.

\[
B(\mathbf{c}_{-p}) = \max_p \pi_p(c_p, \mathbf{c}_{-p}).
\]

Let \( \mathbf{c}^* = (c_1^*, c_2^*, ..., c_P^*) \) denote the Nash equilibrium of competitive price game. Thus \( \mathbf{c}^* \) must satisfy the following condition

\[
c_p^* = B(\mathbf{c}^*), \forall p,
\]

where \( c_{-p}^* \) denotes the set of best responses for player \( q \), \( q \neq p \).

When the spectrum demand strategy (13) of the cognitive base station is given, to solve the best response function of primary networks, we can substitute (13) into the revenue function for primary networks and rewrite (5) as (16) at the top of the next page.

Therefore, we can differentiate (16) with respect to \( c_p \) as

\[
\frac{\partial \pi_p(c)}{\partial c_q} = \alpha_1 K_p \left( \frac{c(P-2)+1}{(\theta(P-1)+1)} - \frac{c_{p}(P-2)+1}{(\theta(P-1)+1)} \right)
\]

\[
+ \frac{\theta \sum_{q \neq p} \left( \sum_k c_k x_{pk} \eta_{pk} + \sum_i \xi_i x_{pi} \eta_{qi} - c_q \right) (\theta(P-2)+1)}{(1-\theta)(\theta(P-1)+1)}
\]

Let

\[
A(c_{-p}) = \frac{\alpha_1 K_p \left( \frac{c(P-2)+1}{(\theta(P-1)+1)} - \frac{c_{p}(P-2)+1}{(\theta(P-1)+1)} \right)}{(1-\theta)(\theta(P-1)+1)}
\]

and

\[
B = \frac{(\theta(P-2)+1)}{(1-\theta)(\theta(P-1)+1)}.
\]

And let \( \frac{\partial \pi_p(c)}{\partial c_q} = 0 \). Hence, we can rewrite (17) as

\[
\frac{\partial \pi_p(c)}{\partial c_p} = \alpha_1 K_p B - 2c_p B + A(c_{-p}) = 0
\]

Therefore, we can obtain the Nash equilibrium solution \( c_p^* \) by solving the above set of linear equations (20). After obtaining the set of prices \( c^* \) at the Nash equilibrium, the size
of spectrum bought for the secondary network from primary network $p$ can be obtained relying on $w^*_p(c^*)$.

For the prices competition game $G$ among primary networks, it’s necessary to prove the existence and uniqueness of Nash equilibrium. Thus, we give the following theorem.

**Theorem 2 (Existence):** A Nash equilibrium exists in the price competition game $G$ for the primary networks.

*Proof:* Firstly, price $c_p$ is a nonempty convex and compact subset of Euclidean space. For utility function $\pi_p(c)$, we know that the first partial derivative of $\pi_p(c)$ with respect to $c_p$ is (20), and the second partial derivative is $\pi''_p(c) = -2B$. Obviously, $B > 0$, thus $\pi''_p(c) < 0$. Hence $\partial\pi_p(c)/\partial c_p$ is strictly decreasing. And

$$\lim_{c_p\to 0} \frac{\partial\pi_p(c)}{\partial c_p} = \alpha_1 \kappa_B + A(c_{-p}) \geq \alpha_1 \kappa_B + c_p B > 0,$$

$$\lim_{c_p\to \infty} \frac{\partial\pi_p(c)}{\partial c_p} = \alpha_1 \kappa_B - 2c_p B + A(c_{-p}) = -\infty < 0,$$

thus utility function $\pi_p(c)$ is first strictly increasing, and then strictly decreasing. This means that utility function $\pi_p(c)$ is a concave function. We know that a concave function is also strictly quasiconcave. Hence, based on [23], there exists a Nash equilibrium for the price competition game $G$.

**Theorem 3 (Uniqueness):** The price competition game $G$ for the primary networks has a unique Nash equilibrium.

*Proof:* From Theorem 2, we know that there exists a Nash equilibrium for the price competition game $G$. Let $c^*$ be the Nash equilibrium, which must satisfy the best response function (14). Therefore, we only need to prove that $B(c^*) = (\alpha_1 \kappa_B + A(c^*)) / 2B$ is a standard function. A function is said to be a standard function when the following properties are satisfied [24]:

1. Positivity: $B(c^*) > 0$; 2. Monotonicity: if $c^*_p > (c^*_p)'$, then $B(c^*_p) \geq B((c^*_p)')$; 3. Scalability: for all $\lambda > 1$, $\lambda B(c^*_p) > B(\lambda c^*_p)$

The positivity property is obviously satisfied for the best response function $B(c^*_p)$. To prove the monotonicity, we know that $A(c^*_p)$ in (18) is an increasing function of $c^*_p$.

Thus when $c^*_p > (c^*_p)'$, we have $A(c^*_p) > A((c^*_p)')$, which is equal to $B(c^*_p) \geq B((c^*_p)')$. Therefore, the monotonicity is proved. For the scalability, for all $\lambda > 1$, we have

$$\lambda B(c^*_p) - B(\lambda c^*_p) = \lambda \alpha_1 \kappa_B + A(c^*_p) = \alpha_1 \kappa_B + A((\lambda c^*_p))$$

$$\lambda = \frac{(\lambda - 1) \kappa_B}{2B}.$$ (23)

From (18), we know that

$$\lambda A(c^*_p) - A(\lambda c^*_p) = \frac{(\lambda - 1)(\sum c_k x_{pk} \eta_{pq} + \sum c_k x_{p} \eta_{p})}{(1-\theta)(P-1)+1} \geq 0.$$ (24)

From (13), we have $0 \leq w^*_p \leq W_p$.

Let

$$\Gamma = \left[ \frac{\sum c_k x_{pk} \eta_{pq}}{(1-\theta)(P-1)+1} \right]$$

and

$$\theta \sum_{q \neq p} \left[ \frac{\sum c_k x_{pk} \eta_{pq}}{} \right],$$

thus we can get

$$0 \leq -c_p (\theta (P-2) + 1) + \theta \sum_{q \neq p} c_q \leq W_p (1-\theta) (\theta (P-1)+1).$$

Obviously, $\Gamma \geq 0$ must be satisfied. Otherwise, we can prove (26) is contradict if $\Gamma < 0$. To satisfy (26) when $\Gamma < 0$, we must have

$$\theta \sum_{q \neq p} c_q \geq c_p (\theta (P-2) + 1) \geq -\Gamma > 0.$$ (27)

To satisfy (27), if and only if $\theta = 1$ and $c_q = c_p, \forall q, p$ are satisfied. In this case, we can obtain

$$\Gamma = -c_p (\theta (P-2) + 1) + \theta \sum_{q \neq p} c_q = -\Gamma < 0,$$ (28)

which is contradicted with (26). Thus we must have $\Gamma \geq 0$. Hence we have $\lambda A(c^*_p) - A(\lambda c^*_p) \geq 0$.

Therefore, substitute (24) into (23), we could get the inequality (29) at the top of next page. Thus, the scalability of
best response function is proved. The best response function of price competition game $G$ is a standard function. Then based on [24], the fixed point $c^*$ is unique for a standard function. Therefore, the Nash equilibrium of the price competition game is unique.

D. Gradient Based Iteration Algorithm to Obtain the Stackelberg Equilibrium

Give the backward induction to solve the three-stage Stackelberg game in the above subsection, in this Subsection, we propose a gradient based iteration algorithm to get the Stackelberg equilibrium.

First, we can analyze the three stage Stackelberg equilibrium using the Subgame Perfect Equilibrium (SPE), which is general determined by backward induction method. Based on the above analysis, we know that each stage exists an unique Nash equilibrium. Therefore, there exists a Stackelberg equilibrium for the three-stage Stackelberg game, and the equilibrium is unique.

Next, we can use the following gradient based iteration algorithm to obtain the three-stage Stackelberg game equilibrium.

Algorithm 1: Gradient Iteration Algorithm for Stackelberg Equilibrium

1. Initialization: primary network $p$ randomly offers a spectrum price $c_p$.
2. Repeat the iteration
   (a). Do the energy-efficient power allocation for FBSs or MSUs, and the cognitive base station decides the spectrum $w_p$ and allocates it to FBSs or MSUs to maximize its revenue (3)
   (b). The primary networks update the prices as
   $$ c(t + 1) = c(t) + \mu \nabla \pi(c(t)) \quad (30) $$
   (c). Until $\|c(t) - c(t - 1)\| / \|c(t - 1)\| \leq \varepsilon$ end iteration.

Here, $\mu$ is the iterative step size of the price, $\pi = \{\pi_1, ..., \pi_P\}$, $\nabla \pi(c(t))$ is the gradient with $\partial \pi_p(c(t)) / \partial c_p$.

In the proposed algorithm, for each iteration, the cognitive base station can decide the spectrum demand and allocate the spectrum to FBSs or MSUs when the prices are updated. Then until the prices are converged, the algorithm is stopped. The algorithm proposed above can get the Stackelberg game equilibrium when the prices are converged, which will be shown in the simulation part of this paper.

In practical networks, the proposed gradient-based iteration algorithm to obtain the three-stage Stackelberg game equilibrium can be implemented as follows:

1. The primary networks first randomly offer spectrum selling prices and broadcast the information.
2. Then the cognitive base station receives the price information, and notices the number of primary networks to FBSs and MSUs, then FBSs and MSUs feedback the channel state information to the cognitive base station.
3. The cognitive base station decides to buy the size of spectrum and allocates the spectrum to FBSs or MSUs.
4. Then the primary networks update the prices and repeat steps (2), (3) until the prices are converged.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we use computer simulations to evaluate the performance of the energy-efficient resource allocation scheme. For ease of illustration, we first consider a simple network scenario with two primary networks and two femtocells, and there is one user accessed in each femtocell. Then, we consider multiple primary networks, multiple femtocells, and multiple users later. The simulation parameters are set as follows. The total spectrum licensed by each primary network is $25 MHz$, the spectrum efficiency of primary networks is 2 and the design parameter $\alpha_S = 1$. The coverage radius of the cognitive base station is $500m$, and the coverage radius of the femtocell is $20m$. Users randomly locate in the cell. The revenue for the FBS is $c_b = 3$, and the cost of using spectrum is $c_b = 1$, the additional circuit power consumption is $p_a = 0.1W$.

Fig. 3 shows the convergence of the proposed gradient based iteration algorithm for Stackelberg equilibrium. We study the performance of prices determination and spectrum demand over iteration step. From Fig. 3(a), we can observe that given the initial prices offered by primary networks, the price competition game between primary networks can get a Nash equilibrium after several iteration steps. As the price offered by primary network 1 decreases, the spectrum demand from primary network 1 increases. Similarly, the spectrum demand from primary network 2 decreases when the price offered by primary network 2 increases. When the prices offered by primary networks get a Nash equilibrium, the spectrum demand for the cognitive base station also gets a Nash equilibrium in Fig. 3(b).

For the primary networks, there is a price competition game $G$, which means that the primary network should take the prices offered by others into account when determining its price to maximize the revenue. In Fig. 4, we evaluate the performance of the revenue of a primary network when the prices of other primary networks are given. From the
figure, we can see that the revenue of primary network 1 first increases with its price increasing. Then when the price reaches a certain value, the revenue of the primary network begins to decrease. As the prices offered by other primary networks increase, the revenue of primary network 1 increases. This is because that the cognitive base station will buy more spectrum from the primary network when the other primary networks have higher prices. Also we can see that as the prices of other primary networks increase, the primary network can offer a higher price to get the Nash equilibrium, which makes the best response of the primary network has a higher price, and can bring a higher revenue.

In Fig. 5, we analyze the relationship between the spectrum demand from the cognitive base station and the prices offered by primary networks. We can observe that the spectrum demand is a linear function of the prices, which is also shown from our theory analysis of the spectrum demand in (13). We can see that when the price offered by primary network 1 increases, the spectrum from the primary network 1 bought by the cognitive base station decreases. When the price offered by primary network 2 increases, there will be more spectrum from primary network 1 bought by the cognitive base station.

The performance of FBS in terms of its energy efficiency over the power budget is evaluated in Fig. 6. From the figure, we can observe that the energy efficiency first increases with its power increasing, then when the power reaches a certain point, the energy efficiency begins to decrease. This is because that there is a tradeoff between transmission capacity and power consumption for the energy-efficient power allocation. The better the channel condition, the more
the energy efficiency obtained in the FBS. Also, we can see from the figure that less power is needed for the FBS with good channel condition to obtain the same energy efficiency for the FBS.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we have studied the issues of spectrum sharing and resource allocation for heterogeneous cognitive radio networks with femtocells to improve energy efficiency. We have formulated the resource allocation problem as a three-stage Stackelberg game. In Stage I, the primary network offers the spectrum selling price to the cognitive base station. In Stage II, the cognitive base station decides to buy the spectrum size from a primary network and allocates the spectrum to femtocells or macro secondary users. In Stage III, the femtocell base station perform power allocation for the femtocell secondary users. Then we have used the backward induction method to solve the resource allocation and proved the existence and uniqueness of the Stackelberg game equilibrium. A gradient based iteration algorithm has been proposed to obtain the Stackelberg equilibrium solution. Simulation results have been presented to demonstrate the performance of the proposed scheme.

In practice, it is difficult to have the perfect knowledge of a dynamic channel. In our future work, we will consider imperfect channel state information in energy-efficient resource allocation for heterogeneous cognitive radio networks with femtocells.

REFERENCES